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## Essays on Asymmetric Contests and Urbanization in India

Pulkit K. Nigam

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ESSAYS ON ASYMMETRIC CONTESTS AND URBANIZATION IN INDIA

by

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## ABSTRACT

I study asymmetric all-pay auction contests where the prize has the same value for all players, but players might have different cost functions. I allow for the cost functions to be discontinuous as long as they are right-continuous. In that setting, I determine sufficient conditions for existence and uniqueness of the conventional mixed-strategy equilibrium. Employing this framework, I discuss the implementation of a soft cap on bids and the effect that has on the conventional mixed-strategy equilibrium and players' bidding behavior, especially with respect to a situation where there is no cap on bids. I also determine the total cost and expected aggregate bids which would influence, and also have an effect on the organizing of such contests.

Drawing from the framework mentioned above, I analyze the implementation of a rigid cap on bids. Rigid cap being one which simply cannot be breached. I determine the players' bidding behavior in the conventional mixed-strategy equilibrium and compute the total cost and expected aggregate bids in this situation.

In the fourth chapter, I explore linguistic explanations for the extremely low labor mobility, but paradoxically high urban wage premium in India. I show how linguistic diversity in India hinders internal migration across state borders. I also find evidence, albeit a weak one, to show that an individual who can speak English is more likely to migrate to an urban center. I find much stronger evidence that links educational attainment with migrating to urban centers.

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## LIST OF SYMBOLS

$x_i$	bid amount for player $i$ .
$V$	Value of prize, same for both players.
$C_i(x)$	Cost function faced by player $i$ . Cost is a function of the bid amount $x$
$L_i(x)$	Cost function faced by player $i$ up till the point of discontinuity.
$F_i$	cumulative distribution function for player $i$ .
$s$	soft cap on bids, $s > 0$ .
$m$	monetary penalty $m > 0$ , or fine imposed on the player who breaches the soft cap.
$r$	rigid cap on bids, $r > 0$ .



## LIST OF ABBREVIATIONS

IRR.....	Incident Rate Ratio
NE .....	Nash Equilibrium
RAB .....	Aggregate Bids in case of Rigid Cap on bids
RTC.....	Total Cost in case of Rigid Cap on bids
SAB.....	Aggregate Bids in case of Soft Cap on bids
STC .....	Total Cost in case of Soft Cap on bids

# CHAPTER 1

## INTRODUCTION

Contests are events where two or more interested parties or players, compete by expending effort or resources in order to secure something of value to all players. Contest can be symmetric, or asymmetric, for a variety of reasons. There could be player specific attributes that lead one player to have a certain advantage and thereby making the contest uneven, and asymmetric. In such contests, a cap on bids is placed so as to limit the spending of the players, so that no player can gain any advantage over the other. There are many examples of such situations where a cap on spending is implemented with the view of making the contest fairer. For example, political campaign financing laws in many countries ensure that the money that any candidate receives to fight an election campaign, and how much she spends during the campaign is monitored very tightly so that there is no expenditure that is unaccounted for.

Che and Gale (1998), were perhaps the first to study caps being put in place in a game theoretic setting of asymmetric contests. Che and Gale (1998) however, considered a cap which could not be breached by anyone. They showed that the stronger of the two players in the contest would see her chances of winning this contest to reduce slightly, and increasing the total cost. Kaplan and Wettstein (2006) pointed out that in most practical situations, a cap on bids is not one that cannot be breached. An example could be one of breaching the speed limit while driving. In such cases in the real world, breaching a cap attracts a fine, monetary or otherwise. Kaplan and Wettstein (2006) show that a contest

with no cap on bids stochastically dominates one with cap on bids. We study the three possible cases here, one with a ‘soft’ cap on bids, which has an associated fine for breaching the cap, the second case with no cap on bids, and finally one with rigid cap on bids which cannot be breached. Additionally, we also model the contest in a manner distinct from earlier approaches in the literature, in that, we allow for players to face discontinuous cost functions, which competing for a single prize of homogenous valuation. With discontinuous cost functions, we attempt to capture the jump in cost that a fine, or a monetary penalty could potentially create if a player were to breach the cap. Alternatively, a player might factor the jump in cost due to the fine, and bid accordingly.

The third chapter, unlike the first two, which deal with game theoretic models, is about internal migration and urbanization in India. As India grows economically and also in terms of its population soon to become the world’s largest, internal migration and urbanization in India will be of greater interest. I have tried to bring to fore the aspect of linguistic diversity in India, which is unlike any other country, and how that impact internal migrations and urbanization. This draws on the cross-country done by Chauvin et. al. (2017) who find it difficult to explain the low (labor) mobility in India and the high urban wage premiums there. I attempt to find answers to the questions raised by Chauvin et. al. (2017) in the linguistic diversity that exists in India. Despite issues pertaining to the availability of satisfactory data, I am able to show and highlight the impact that linguistic diversity in India has on internal migration in India.

## CHAPTER 2

### ASYMMETRIC CONTESTS WITH A SOFT CAP ON BIDS

#### 1. Introduction

A cap on spending has conventionally been argued for as a way of providing a level playing field to contestants of varying capabilities. When the contest is an election campaign putting a cap on how much a political party, or a candidate can spend on the campaign is meant to ensure that no one party or candidate accumulates advantage based simply on their capacity to outspend their opponent(s). Similarly, a cap on salary paid to players by sports teams is meant to ensure that the best talent is not concentrated in just a handful of teams making them extraordinarily dominant in a sports league. Usually these caps are enforced through fines or penalties levied upon those who spend in excess of the cap. For the 2015 season, the Major League Baseball (MLB) had a spending cap in form of a Competitive Balance Tax, also known as a Luxury Tax, where a team spending more than \$189 million in player payroll would have to pay a tax for every dollar they were above the cap<sup>1</sup>. This however did not prevent Los Angeles Dodgers from spending in excess of \$298 million, significantly above the \$189 million cap. LA Dodgers was not alone in this, they were joined by New York Yankees, Boston Red Sox, and San Francisco

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<sup>1</sup> Breaking Down Over \$400 Million In MLB Luxury Tax Penalties Since 2003. Maury Brown, Forbes, Dec 3, 2015

<https://www.forbes.com/sites/maurybrown/2015/12/03/breaking-down-over-400-million-in-mlb-luxury-tax-penalties-since-2003/#33bb1623541a>

Giants. Owing to the structure of this Luxury Tax, the amount that these teams had to pay as a consequence of breaching the cap ranges from \$1.3 million for SF Giants to almost \$44 million for LA Dodgers. In the field of politics, ‘Vote Leave’, the official campaign group advocating British exit from the European Union in the 2016 referendum on that issue, was fined £61,000 by the (UK) Electoral Commission for breaching the £7 million spending limit, and spending approximately an extra £500,000<sup>2</sup>. More recently, soccer’s European governing body, UEFA, opened investigation into accusations of violation of Financial Fair Play rules by the English team Manchester City and its owner<sup>3</sup>. These are just some of the instances that demonstrate that contestants are willing to exceed a cap on spending and include the imminent penalty as part of the costs in pursuit of maximizing their payoffs<sup>4</sup>.

Given the examples discussed above, it is imperative to point out that there is a distinction between the cost that players incur in a contest, and the bids they make. The organizer may want players to incur a high cost in preparing for a contest as a signal of their intent. For example, in a tendering process for a contract, the organizing party (government agency or private firm) may want prospective contractors to come up elaborate proposals for implementation of the project under consideration. This might increase the cost for the prospective contractors, the organizing party could use the signal

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<sup>2</sup> Vote Leave fined and referred to police. Jim Pickard and Camilla Hodgson, Financial Times, July 17, 2018 <https://www.ft.com/content/a8b848ce-8987-11e8-b18d-0181731a0340>

<sup>3</sup> Manchester City accuse Uefa of leaks amid Champions League ban threat. David Conn, The Guardian, May 14, 2019 <https://www.theguardian.com/football/2019/may/14/manchester-city-uefa-champions-league-ban-protest-innocence>

<sup>4</sup> In the case of MLB 2015 season described above, the LA Dodgers spent an extra \$109 million above the cap, which came at a cost of almost \$44 million, this shows that costs beyond the threshold of a cap, ‘jumps’ owing to the penalty imposed.

to reject those candidates who may come across as tacky. Alternately, in a promotion contest to be a top executive, a firm might prefer to lower costs for all players and simply expect that the prospective candidates would put in maximum effort as their bid. This shows that the distinction between the cost to players (aggregate cost) and bids by players (aggregate bids) may matter to both the players and the organizer.

## **2. Literature**

Che and Gale (1998) was amongst the first papers to formally analyze model a contest with caps. They model a contest as a complete information two-player all-pay auction where players have different valuations for the same prize and their bid is the cost they incur in participating in the contest. They consider a rigid cap on bids which simply cannot be breached. As their model considers the cost to be the bid, there is no distinction between aggregate cost, and aggregate bids. They find that a small rigid cap on bids leads to an increases in aggregate cost in the contest, which is the same as an increase in aggregate bids as per their model. Kaplan and Wettstein (2006) in response point out that more often than not, there is a fine or a penalty associated with the breaching of a cap on bids, and so a soft, and not a rigid cap is more commonplace. They model the costs incurred by players as strictly increasing continuous functions of the bid amount. They argue that bidding without cap stochastically dominates bidding with a (soft) cap, and that the imposition of a soft cap does not affect aggregate costs but always reduces expected bids. They further argue that monetary fines could be welfare enhancing, while non-monetary fines like banning a team may be detrimental. In their response, Che and Gale (2006) discuss and argue that the aggregate costs increase due cost-equalizing shifts when players face different cost functions. They further show that monetary fines have no effect on

expected aggregate cost, while a nonmonetary penalty generates strictly higher expected aggregate cost. In a more recent paper, Olszewski and Siegel (2019) look at the effect of both, rigid and soft caps on the aggregate costs incurred by, and aggregate bids made by contestants in large all-pay auction contests; where a large number of heterogeneous contestants compete for a large number of prizes. They conclude that as far as aggregate costs are concerned, flexible caps have next to no effect, but rigid caps always lower aggregate costs. Rigid caps also decrease aggregate bids when cost functions are linear or concave, but could increase aggregate bids for convex cost functions in some conditions. Flexible caps on the other hand, always decrease aggregate bids. Their approach is different from the studies mentioned earlier, in that, they consider a situation with  $n$  number of players and  $n$  number of prizes, while earlier studies are 2-player games with a single prize and 2-valuations of that prize. Furthermore, Olszewski and Siegel (2019) use an assortative allocation to award  $n$  prizes to  $n$  players based on their bid. The authors do have a similarity with earlier works as each player faces strictly increasing and continuous cost functions under all circumstances. These studies show there to be some ambiguity around the effects of a cap on bids<sup>5, 6</sup>. In this study we focus on soft cap on bids, where a monetary penalty is implemented for breaching a cap on bids. Our study sits well with the discussion initiated by Che and Gale (1998) and carried forward by Kaplan and Wettstein (2006), and Che and Gale (2006). Our findings support Kaplan and Wettstein (2006) when they say that bidding without a cap strictly dominates bidding with a soft cap on bids. We further are in

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<sup>5</sup> Pastine and Pastine (2013) extended Kaplan and Wettstein (2006) and Che and Gale (2006) by incorporating politician preferences into their framework.

<sup>6</sup> Other studies look at the Caps in Contests from different angles. Gavious, Moldovanu, and Sela (2002) study private-information contests with caps. Szech (2015) revisits Che and Gale (1998) and analyzes different tie-breaking rules.

agreement with Kaplan and Wettstein (2006), and Olszewski and Siegel (2019), in so far as the decrease in aggregate bids is concerned when a soft cap is implemented. However, we show that aggregate costs decrease when a soft cap with a monetary fine is implemented, while both, Kaplan and Wettstein (2006), and Olszewski and Siegel (2019), argue that a soft cap has no effect on aggregate costs. These results are highlighted in Table 1.1.

We model an asymmetric contest as an all-pay auction where the players have the same valuation of the prize, but different right-continuous cost functions. In doing so, we seek to capture the ‘jump’, or the discontinuity in the costs owing to a non-rigid fine or a soft cap being implemented.

It is standard in the literature to model asymmetric contests as all-pay auctions where players have different valuation of a single prize and the same linear cost functions, and to obtain conventional mixed-strategy equilibria, where the most efficient player obtains a positive expected payoff, and the other players get expected zero payoffs. See, for example, Hillman and Riley (1989), Baye, Kovenock, and De Vries (1993, 1996).<sup>7</sup> A typical assumption in this literature is that cost functions are twice continuously differentiable.

We provide sufficient conditions for uniqueness of the conventional mixed-strategy equilibrium in our setting. This equilibrium is qualitatively different from equilibria in previous studies. Moreover, we construct the conventional mixed-strategy equilibrium even if different players have caps at different points and the number of such caps is finite.

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<sup>7</sup> Siegel (2009, 2010, 2014) considers general contests with continuous cost functions.



First, we analyze two-player asymmetric contests. We show that there exists the conventional mixed-strategy equilibrium. However, there might be pure-strategy equilibria in our model, if the assumption about the right-continuity of the cost functions is violated, Example 2 illustrates that situation.

### 3. Asymmetric Two-Player Contest with a Soft Cap

Suppose that two players contest a single prize  $V > 0$ . The prize value is same for both players, but the players' cost functions, indicating their ability, can be different. Thus we have an asymmetric contest.

We define a 'soft' cap on bids as the maximum bid permissible with a penalty imposed on any player who bids in excess of the soft cap. Essentially, a soft cap on bids is one where a player *can* bid in excess of the cap, but at an additional cost incurred in terms of a monetary penalty or fine. Each player  $i$  can bid a positive amount  $x_i < s$ , where  $s > 0$  is the soft cap on bids. For a bid  $x_i \geq s$ , player  $i$  will be penalized and will have to pay a fixed monetary fine  $m \geq 0$ . The imposition of the monetary fine will have the effect of creating a 'jump' in the cost faced by any player. Essentially, the cost function faced by player  $i$  will increase continuously as long as  $x_i < s$ . For  $x_i \geq s$  however, the cost function would be displaced vertically upwards, or jump up by  $m \geq 0$ . We model this asymmetric contest with a soft cap on bids below.

We assume that players have right-continuous cost functions with at most one discontinuity which satisfy the following conditions<sup>8</sup>

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<sup>8</sup> We will discuss left-continuous cost functions below.

$$C_i(x) = \begin{cases} L_i(x), & \text{if } x < s, \\ L_i(x) + m, & \text{if } x \geq s, \end{cases} \quad (1)$$

where,

$$m \geq 0 \quad (2)$$

$$L_1(0) = L_2(0) = 0 \quad (3)$$

$$L_i(x) \text{ is strictly increasing functions for } i = 1, 2, \quad (4)$$

We assume that there exists  $0 < t_i < \infty$  such that

$$C_i(t_i) = V \text{ for } i = 1, 2, \quad (5)$$

and

$$t_2 \leq t_1. \quad (6)$$

based on which, we call Player 1 *more efficient* player.

Each player  $i$  exerts effort  $x_i \geq 0$  in order to win the prize in the all-pay auction and obtains the following payoff

$$u_i(x_1, x_2) = \begin{cases} -C_i(x_i), & \text{if } x_i < x_{-i}, \\ \frac{V}{2} - C_i(x_i), & \text{if } x_1 = x_2, \\ V - C_i(x_i), & \text{if } x_i > x_{-i}. \end{cases} \quad (7)$$

We can describe a mixed-strategy equilibrium in our setting now.

**Theorem 1.** *If conditions (1)–(6) hold, and  $d < t_2$ , then there exists a conventional mixed-strategy NE in the asymmetric two-player contest, where Player 1 randomizes according to the following cumulative distribution function*

$$F_1(x) = \frac{1}{V} C_2(x), \quad (8)$$

*on the interval  $[0, t_2]$  placing an atom at  $x = s$ , if  $s < t_2$ ; and player 2 randomizes according to the following cumulative distribution function, and Player 2 randomizes according to the following cumulative distribution function*

$$F_2(x) = \frac{V - C_1(t_2)}{V} + \frac{1}{V} C_1(x), \quad (9)$$

*on the interval  $[0, t_2]$  placing an atom at zero and at  $x = s$  if  $s < t_2$ .*

Note that if  $m = 0$ , or there is no penalty gap, then we get a conventional mixed-strategy NE, similar to the mixed-strategy equilibrium in Hillman and Riley (1989), and in Baye, Kovenock, and De Vries (1996).

**Corollary 1.** *If  $m = 0$  and conditions (3)–(6) hold, then there exists a conventional mixed-strategy NE in the asymmetric two-player contest, where player 1 randomizes according to the following cumulative distribution function*

*Player 1 randomizes according to the following cumulative distribution function*

$$F_1(x) = \frac{1}{V} C_2(x)$$

*on the interval  $[0, t_2]$ , and Player 2 randomizes according to the following cumulative distribution function*

$$F_2(x) = \frac{V - C_1(t_2)}{V} + \frac{1}{V} C_1(x)$$

on the interval  $[0, t_2]$ , placing an atom of size at zero.

The following corollary is a well-known result in the contest literature where cost functions are linear.

**Corollary 2.** *If  $m = 0$ , and*

$$C_1(x) = C_2(x) = x,$$

*then there exists a unique (conventional) mixed-strategy NE in the symmetric two-player contest, where both players randomize according to the following cumulative distribution function*

$$F_1(x) = F_2(x) = \frac{x}{V}$$

on the interval  $[0, t_2]$ .

If  $m > 0$  and  $s < t_2$ , then mass points appear at  $x = 0$  for player 2 and at the point of discontinuity,  $x = s$ , for both players in the conventional mixed-strategy NE. Note that player 1 (2) cannot take advantage of the mass point at  $x = s$  in the cdf of player 2 (1) because her own cost is discontinuous exactly at that point. The following example illustrates Theorem 1.

**Example 1.** Suppose that  $V = 2$ ,  $s = 1$ ,  $m = 0.5$ , the cost functions are

$$C_1(x) = \begin{cases} \sqrt{x}, & \text{if } x < 1, \\ \sqrt{x} + 0.5, & \text{if } x \geq 1, \end{cases}$$

and

$$C_2(x) = \begin{cases} x, & \text{if } x < 1, \\ x + 0.5, & \text{if } x \geq 1. \end{cases}$$

Then,  $t_2 = 1.5 < 2.25 = t_1$  and conditions (1) – (6) hold. In the conventional mixed-strategy NE, player 1 randomizes according to the following cumulative distribution function:

$$F_1(x) = \begin{cases} \frac{1}{2}x, & \text{if } x \in [0, 1), \\ \frac{1}{2}x + \frac{1}{4}, & \text{if } x \in [1, 1.5], \end{cases}$$

on the interval  $[0, 1.5]$  placing an atom of size  $\frac{1}{4}$  at one. Player 2 randomizes according to the following cumulative distribution function

$$F_2(x) = \begin{cases} \frac{1.5 - \sqrt{1.5}}{2} + \frac{1}{2}\sqrt{x}, & \text{if } x \in [0, 1) \\ \frac{2 - \sqrt{1.5}}{2} + \frac{1}{2}\sqrt{x}, & \text{if } x \in [1, 1.5] \end{cases}$$

on the interval  $[0, 1.5]$ , placing an atom of size  $\frac{1}{4}$  at one and an atom of size  $\left(\frac{1.5 - \sqrt{1.5}}{2}\right)$  at zero.

Figure 2.1 below shows the cost functions used in this example on the left-hand side panel while the cumulative distribution functions calculated above are plotted on the right-hand side panel. The solid line represents Player 1's cost function and cumulative distribution function in the respective panels, while the dotted line represents the same for Player 2 in the respective panels.

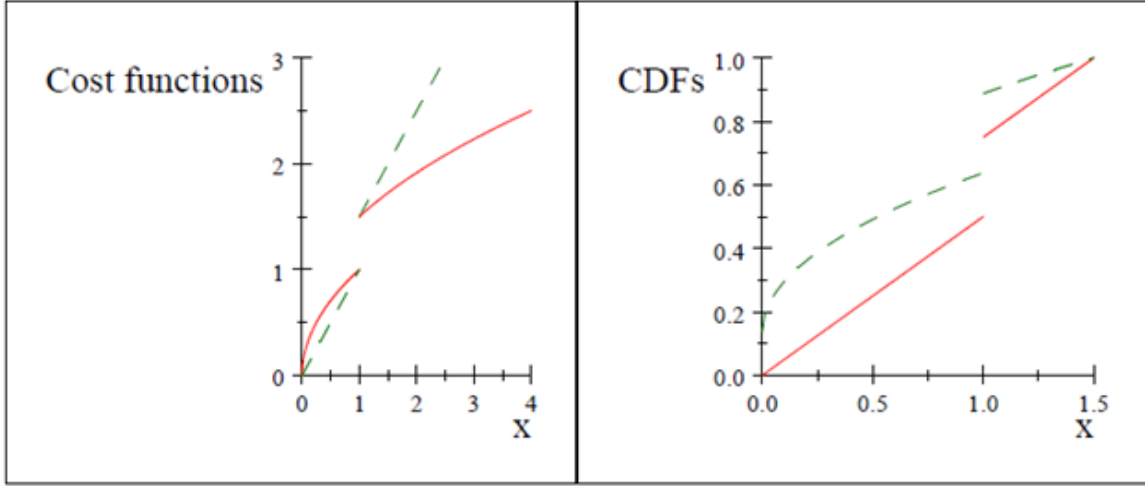


Figure 2.1 Cost Functions and Cumulative Distribution Functions

$C_1(x)$  – solid line and  $C_2(x)$  – dash line on the left panel

$F_1(x)$  – solid line and  $F_2(x)$  – dash line on the right panel

Next, we establish uniqueness of the conventional mixed-strategy NE in the asymmetric two-player contest.

**Theorem 2** *If conditions (1) – (6) hold, then the conventional mixed-strategy equilibrium is a unique NE in the asymmetric two-player contest.*

The proof is standard and similar to the proofs of Propositions 1 and 2 in Hillman and Riley (1989) and thus is omitted. The following example shows that there can be a pure-strategy NE if cost functions are left-continuous, or assumption (1) is violated.

**Example 2.** Suppose that  $V = 1$ ,  $s = 0.1$ , and  $m = 0.9$ , and

$$C_1(x) = C_2(x) = \begin{cases} 0.1x, & \text{if } x \in [0, 0.1], \\ 0.1x + 0.9, & \text{if } x \in (0.1, 1], \end{cases}$$

Then,  $t_2 = 1 = t_1$ , and conditions (2) – (6) hold.

Note that there exists a pure strategy equilibrium where both players bid

$$x_1 = x_2 = 0.1$$

and obtain expected payoffs

$$E\pi_i(0.1, 0.1) = 0.5 - 0.01 = 0.49, \text{ for } i = 1, 2.$$

### 3.1 Aggregate Bids and Total Cost

Having determined the bidding behavior for the two players, we now focus on the total cost in the asymmetric two-player contest with a soft cap. As discussed earlier, the total cost, and the expected aggregate bids, as given below, are also important aspects of an asymmetric contest.

$$STC = \int_0^{t_2} C_1(x) dF_1(x) + \int_0^{t_2} C_2(x) dF_2(x),$$

and the expected aggregate bids

$$SAB = \int_0^{t_2} x dF_1(x) + \int_0^{t_2} x dF_2(x).$$

From Theorem 1,

$$\int_0^{t_2} C_1(x) dF_1(x) = \begin{cases} \frac{1}{V} \int_0^{t_2} L_1(x) dL_2(x), & \text{if } t_2 < s, \\ \frac{1}{V} \int_0^s L_1(x) dL_2(x) + \frac{1}{V} \int_s^{t_2} (L_1(x) + m) dL_2(x), & \text{if } t_2 \geq s, \end{cases} \quad (10)$$

and

$$\int_0^{t_2} C_2(x) dF_2(x) = \begin{cases} \frac{1}{V} \int_0^{t_2} L_2(x) dL_1(x), & \text{if } t_2 < s, \\ \frac{1}{V} \int_0^s L_2(x) dL_1(x) + \frac{1}{V} \int_s^{t_2} (L_2(x) + m) dL_1(x), & \text{if } t_2 \geq s. \end{cases} \quad (11)$$

Therefore, we get the following result.

**Theorem 3** *The total cost is*

$$STC = \begin{cases} C_1(t_2), & \text{if } t_2 < s \\ C_1(t_2) - \frac{m}{V}(L_1(s) + L_2(s) + m), & \text{if } t_2 \geq s \end{cases} \quad (12)$$

*The total cost is maximized, if there is no soft cap, or  $m = 0$ . The expected aggregate bids are independent from the soft cap,*

$$SAB = \frac{1}{V} \left( t_2(L_1(t_2) + L_2(t_2)) - \left( \int_0^{t_2} L_1(x) dx + \int_0^{t_2} L_2(x) dx \right) \right). \quad (13)$$

The following example illustrates the theorem.

**Example 3.** Suppose that  $V = 2$ ,  $m = 0$ ,  $C_1(x) = \sqrt{x}$ , and  $C_2(x) = x$ .

Then,  $t_2 = 2 < 4 = t_1$  and conditions (1) – (6) hold. From Corollary 2, in the conventional mixed-strategy NE, Player 1 randomizes according to the following cumulative distribution function

$$F_1(x) = \frac{1}{2}x,$$

on the interval  $[0, 2]$ , and Player 2 randomizes according to the following cumulative distribution function

$$F_2(x) = \frac{2 - \sqrt{2}}{2} + \frac{1}{2}\sqrt{x}$$

on the interval  $[0, 2]$ , placing an atom of size  $\left(\frac{2 - \sqrt{2}}{2}\right)$  at zero. From (12), the total cost is

$$STC = C_1(t_2) = \sqrt{2} \approx 1.41.$$



From (13), the expected aggregate bids are

$$\begin{aligned}
SAB &= \frac{1}{V} \left( \int_0^{t_2} x dL_2(x) + \int_0^{t_2} x dL_1(x) \right) \\
&= \frac{1}{V} \left( t_2 (L_1(t_2) + L_2(t_2)) - \left( \int_0^{t_2} L_1(x) dx + \int_0^{t_2} L_2(x) dx \right) \right) \\
SAB &= \frac{1}{2} \left( 2(\sqrt{2} + 2) - \left( \frac{2}{3} 2^{\left(\frac{3}{2}\right)} + \frac{2^2}{2} \right) \right) \approx 1.47.
\end{aligned}$$

Now, suppose that a soft cap is introduced. From Example 1,  $m = 0.5$ , and  $s = 1$ ,

$$C_1(x) = \begin{cases} \sqrt{x}, & \text{if } x < 1, \\ \sqrt{x} + 0.5, & \text{if } x \geq 1, \end{cases}$$

and

$$C_2(x) = \begin{cases} x, & \text{if } x < 1, \\ x + 0.5, & \text{if } x \geq 1. \end{cases}$$

Then,  $t_2 = 1.5 < 2.25 = t_1$ . and conditions (1) – (6) hold. In the conventional mixed-strategy NE, Player 1 randomizes according to the following cumulative distribution function

$$F_1(x) = \begin{cases} \frac{1}{2}x, & \text{if } x \in [0, 1), \\ \frac{1}{2}x + \frac{1}{4}, & \text{if } x \in [1, 1.5], \end{cases}$$

on the interval  $[0, 1.5]$  placing an atom of size  $\frac{1}{4}$  at one. Player 2 randomizes according to the following cumulative distribution function

$$F_2(x) = \begin{cases} \frac{1.5 - \sqrt{1.5}}{2} + \frac{1}{2}\sqrt{x}, & \text{if } x \in [0, 1), \\ \frac{2 - \sqrt{1.5}}{2} + \frac{1}{2}\sqrt{x}, & \text{if } x \in [1, 1.5], \end{cases}$$

on the interval  $[0, 1.5]$ , placing an atom of size  $\frac{1}{4}$  at 1 and an atom of size  $\left(\frac{1.5 - \sqrt{1.5}}{2}\right)$  at zero.

From (12), the total cost is

$$STC = C_1(t_2) - \frac{m}{V}(L_1(s) + L_2(s) + m),$$

$$STC = \sqrt{1.5} + 0.5 - \frac{0.5}{2}(1 + 1 + 0.5) \approx 1.10.$$

From (13), the expected aggregate bids are

$$\begin{aligned} SAB &= \frac{1}{V} \left( \int_0^{t_2} x dL_2(x) + \int_0^{t_2} x dL_1(x) \right) \\ &= \frac{1}{V} \left( t_2(L_1(t_2) + L_2(t_2)) - \left( \int_0^{t_2} L_1(x) dx + \int_0^{t_2} L_2(x) dx \right) \right) \\ SAB &= \frac{1}{2} \left( 1.5(1.5 + \sqrt{1.5}) - \left( \frac{2}{3} 1.5^{\left(\frac{3}{2}\right)} + \frac{1.5^2}{2} \right) \right) \approx 0.87. \end{aligned}$$

Note here that both, the total cost and aggregate bids decrease when a soft cap is introduced.

## 4. Conclusions

We have shown that a two player asymmetric contest can be modeled as one where the players have the same valuation of the prize, but face different cost functions. This provides an additional approach to modeling such contests compared to the existing approach in the literature. The consistency of our results with those in the existing literature show that our approach is viable and is successful in presenting an additional framework. Additionally, the introduction of discontinuous cost functions in the model, and using them capture the ‘jump’ in costs that a player would face when there is a penalty imposed on breaching a certain spending limit, is another contribution to the literature.

Using our model, we have also explored the various policy options around the imposition of an exogenous cap on bids, which organizers face when conducting such contests. We consider the impact such policy has on the total cost to the players and aggregate bids made by them. We show that, in terms of increasing the total cost faced by the players, a policy of no cap on bids strictly dominates the one with a soft cap on bids with a penalty for breaching the cap. While Kaplan and Wettstein (2006), and Olszewski and Siegel (2019) show that a soft cap has no effect on total cost; Che and Gale (2006) show an increase in total cost for a soft cap with non-monetary penalties. Our findings on the other hand, show that implementing a soft cap on bids would decrease the total cost faced by the players. We believe that our findings are more intuitive as discussed below.

On expected aggregate bids made by players, Example 3 shows that, the aggregate bids reduce upon the implementation of a soft cap on bids. Our findings, in the context of

aggregate bids, support those of Kaplan and Wettstein (2006), and Olszewski and Siegel (2019).

#### *A. Cap on bids and competition*

While analyzing various policies regarding caps on bids in an asymmetric contest, it may be fruitful to generally consider the effect of such policies on how competitive they may, or may not, make the contest itself. With there being no cap on bids, the most efficient (advantaged) player has a clear advantage. Such players may be advantaged in terms of certain reputational, experiential, or financial factors. This would provide the less efficient player with few incentives to bid high, knowing this, the most efficient player will also not bid as high as they could have. There is nonetheless, still a non-zero possibility that the less efficient player could bid high and catch the most efficient player unaware and win the contest. However, when a policy of a soft cap on bids is implemented, so that any player who bids above a certain capped amount would be required to pay a monetary fine, then only the most efficient player would take that the opportunity to bid close to the cap, or even breach the cap. The less efficient player would have less inclination to bid close to the capped amount where they could lose to the more efficient player(s); and have even less of a proclivity to breach the cap. Knowing this, even the more efficient player(s) would then not bid large amounts. As a consequence, both, the expected aggregate bids and the total cost, which is a function of the bids, would therefore decrease in this situation compared to the policy of no cap on bids, where the less efficient player(s) does not need to contend with the possibility of an imminent penalty upon bidding in excess of a specific amount.

### *B. Total Cost and Aggregate Bids*

As has been discussed earlier in the introduction, total cost and aggregate bids may be of interest to both organizers and player. We have shown the effect that a soft cap on bids has on the two. Our results imply that if an organizer would want to lower the cost to players, then they should implement a soft cap on bids, that however, would lead to a decrease in expected aggregate bids. On the other hand, if an organizer would like to increase aggregate bids, then going for a no cap on bids option would be her choice, that however, maximizes the total cost for the players or the participants.

**Table 2.1 Summary of results pertaining to a soft cap on bids.**

This table summarizes our findings with regards to soft cap on bids, and compares it with previous literature on this subject.

	Che and Gale (1998)	Kaplan and Wettstein (2006)	Che and Gale (2006)	Olszewski and Siegel (2019)	This paper
Effect on Aggregate Cost					
Soft Cap	--	No Effect	Monetary fine: No Effect Nonmonetary penalty: Increases	No Effect	Decreases
No Cap	--	--	--	--	Dominates
Effect on Aggregate Bids					
Soft Cap	--	Decreases	--	Decreases	Decreases
No Cap	--	Dominates	--	--	Dominates

## CHAPTER 3

### ASYMMETRIC CONTESTS WITH A RIGID CAP ON BIDS

#### **1. Introduction**

The costs in a contest such as a lobbying or a political campaign can be substantially high at times, where the contestants can be spending large sums in order to win the contest. This has been quite apparent in election campaigns in the United States, where the costs of running a campaign has been increasing over the years. Such increase in campaign spending can be quite wasteful, as it would require a politician to considerably increase fund-raising efforts, which may come at cost of other significant activities. Additionally, a donor could appropriate undue influence on electoral outcomes, or on policy positions by making a large enough campaign contribution (Che and Gale, 1998). Such situations may also give one contestant, an undue advantage over another. Similar situations could also exit in the field of professional team sports. A team may be able to significantly outspend another, and thereby purchase many extremely expensive players, potentially giving them extraordinary advantage over other teams. Despite such a variety of scenarios, a cap on spending has conventionally been argued for as a way of providing a level playing field to contestants of varying capabilities. In this paper we consider a two-player asymmetric contest with a single prize. A rigid cap is placed on the amount that the players can bid.

In addition to their being contestants, contests also tend to have other interested parties whom we may call organizer(s). It is possible, that the organizer(s) might desire for

the contestants to incur a high cost in preparation for a contest. The organizer might use the cost incurred by the contestant as a signal of their intent. A firm wanting to build its new corporate office might prefer various architecture firms in competition for the project, to be as elaborate and detailed in their plans and designs for the new corporate office. Conversely, the organizer might want contestants to incur as low a cost as possible in preparation for a contest, but would prefer the contestants to expend maximum effort during the contest. This is the distinction between aggregate cost, that contestants incur in participating in the contest, and aggregate bids which is the amount or effort that players expend as they compete.

## **2. Literature**

Che and Gale (1998) was amongst the first papers to formally analyze model a contest with caps. They model a contest as a complete information two-player all-pay auction where players have different valuations for the same prize and their bid is the cost they incur in participating in the contest. They consider a rigid cap on bids which simply cannot be breached. As their model considers the cost to be the bid, there is no distinction between aggregate cost, and aggregate bids. They find that a small rigid cap on bids leads to an increases in aggregate cost in the contest, which is the same as an increase in aggregate bids as per their model. Kaplan and Wettstein (2006) in response point out that more often than not, there is a fine or a penalty associated with the breaching of a cap on bids, and so a soft, and not a rigid cap is more commonplace. They model the costs incurred by players as strictly increasing continuous functions of the bid amount. They argue that bidding without cap stochastically dominates bidding with a (soft) cap, and that the imposition of a soft cap does not affect aggregate costs but always reduces expected bids.



They further argue that monetary fines could be welfare enhancing, while non-monetary fines like banning a team may be detrimental. In their response, Che and Gale (2006) discuss and argue that the aggregate costs increase due cost-equalizing shifts when players face different cost functions. They further show that monetary fines have no effect on expected aggregate cost, while a nonmonetary penalty generates strictly higher expected aggregate cost. In a more recent paper, Olszewski and Siegel (2019) look at the effect of both, rigid and soft caps on the aggregate costs incurred by, and aggregate bids made by contestants in large all-pay auction contests; where a large number of heterogeneous contestants compete for a large number of prizes. They conclude that as far as aggregate costs are concerned, flexible caps have next to no effect, but rigid caps always lower aggregate costs. Rigid caps also decrease aggregate bids when cost functions are linear or concave, but could increase aggregate bids for convex cost functions in some conditions. Flexible caps on the other hand, always decrease aggregate bids. Their approach is different from the studies mentioned earlier, in that, they consider a situation with  $n$  number of players and  $n$  number of prizes, while earlier studies are 2-player games with a single prize and 2-valuations of that prize. Furthermore, Olszewski and Siegel (2019) use an assortative allocation to award  $n$  prizes to  $n$  players based on their bid. The authors do have a similarity with earlier works as each player faces strictly increasing and continuous cost functions under all circumstances. These studies show there to be some ambiguity around the effects of a cap on bids<sup>9, 10</sup>. In this study we focus on rigid cap on bids, that cannot be breached.

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<sup>9</sup> Pastine and Pastine (2013) extended Kaplan and Wettstein (2006) and Che and Gale (2006) by incorporating politician preferences into their framework.

<sup>10</sup> Other studies look at the Caps in Contests from different angles. Gavious, Moldovanu, and Sela (2002) study private-information contests with caps. Szech (2015) revisits Che and Gale (1998) and analyzes different tie-breaking rules.

Our study sits well with the discussion initiated by Che and Gale (1998) and carried forward by Kaplan and Wettstein (2006), and Che and Gale (2006). Our findings support Kaplan and Wettstein (2006) when they say that bidding without a cap strictly dominates bidding with a soft cap on bids. We further are in agreement with Kaplan and Wettstein (2006), and Olszewski and Siegel (2019), in so far as the decrease in aggregate bids is concerned when a soft cap is implemented. However, we show that aggregate costs decrease when a soft cap with a monetary fine is implemented, while both, Kaplan and Wettstein (2006), and Olszewski and Siegel (2019), argue that a soft cap has no effect on aggregate costs. These results are highlighted in Table 1.

We model an asymmetric contest as an all-pay auction where the players have the same valuation of the prize, but different right-continuous cost functions. In doing so, we seek to capture the ‘jump’, or the discontinuity in the costs owing to a non-rigid fine or a soft cap being implemented.

It is standard in the literature to model asymmetric contests as all-pay auctions where players have different valuation of a single prize and the same linear cost functions, and to obtain conventional mixed-strategy equilibria, where the most efficient player obtains a positive expected payoff, and the other players get expected zero payoffs. See, for example, Hillman and Riley (1989), Baye, Kovenock, and De Vries (1993, 1996).<sup>11</sup> A typical assumption in this literature is that cost functions are twice continuously differentiable.

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<sup>11</sup> Siegel (2009, 2010, 2014) considers general contests with continuous cost functions.

We provide sufficient conditions for uniqueness of the conventional mixed-strategy equilibrium in our setting. This equilibrium is qualitatively different from equilibria in previous studies. Moreover, we construct the conventional mixed-strategy equilibrium even if different players have caps at different points and the number of such caps is finite.

First, we analyze two-player asymmetric contests. We show that there exists the conventional mixed-strategy equilibrium. However, there might be pure-strategy equilibria in our model, if the assumption about the right-continuity of the cost functions is violated, Example 2 illustrates that situation.

### 3. Asymmetric Two-Player Contest with a Rigid Cap

We will consider a rigid cap on bids now. We define a ‘rigid’ cap on bids as the maximum bid permissible. Each player  $i$  can bid a positive amount  $x_i \leq r$  in the contest, where  $r > 0$  is the rigid cap on bids. Suppose that two players contest a prize  $V > 0$ . The value of the prize is same for both players, but the players’ cost functions can be different and satisfy the following conditions

$$L_1(x) \leq L_2(x), \tag{1}$$

with,

$$L_1(0) = L_2(0) = 0 \tag{2}$$

$$L_i(x) \text{ is strictly increasing functions for } i = 1, 2, \tag{3}$$

We assume that there exists  $0 < t_i < \infty$  such that

$$L_i(t_i) = V \text{ for } i = 1, 2, \tag{4}$$

and

$$t_2 \leq t_1. \quad (5)$$

based on which, we call Player 1 *more efficient* player.

Each player  $i$  exerts effort  $x_i \geq 0$  in order to win the prize in the all-pay auction and obtains the following payoff

$$u_i(x_1, x_2) = \begin{cases} -L_i(x_i), & \text{if } x_i < x_{-i}, \\ \frac{V}{2} - L_i(x_i), & \text{if } x_1 = x_2, \\ V - L_i(x_i), & \text{if } x_i > x_{-i}. \end{cases} \quad (6)$$

In this setting, where (1) – (6) hold, and

$$r < t_2. \quad (7)$$

### 3.1 Small Rigid Cap

It is intuitively clear that both players make every possible effort if the rigid cap on bids is small enough. Denote

$$r_0 = \min \left\{ L_1^{-1} \left( \frac{V}{2} \right), L_2^{-1} \left( \frac{V}{2} \right) \right\} = L_2^{-1} \left( \frac{V}{2} \right) < t_2. \quad (8)$$

We get the following result,

**Theorem 1** *if conditions (1) – (5) and*

$$0 < r \leq r_0 \quad (9)$$

hold, then there exists a pure-strategy NE in which each player submits a bid of  $r$  in the asymmetric two-player contest with a rigid cap.

### 3.2 Large Rigid Cap

Suppose the rigid cap  $r$ , is “large”, and given as

$$r_0 < r < t_2. \quad (10)$$

It is obvious here that both players cannot bid the cap level in an equilibrium as at least one of them will get a negative payoff. We can describe the mixed-strategy in our setting now.

**Theorem 2** *If conditions (1) – (5), and (10) hold, then there exists a mixed-strategy NE in the asymmetric two-player contest with a rigid cap,  $r$ , where Player 1 randomizes according to the following cumulative distribution function.*

$$H_1(x) = \frac{1}{V} L_2(x), \quad (11)$$

*on the interval  $[0, t]$  placing an atom at  $x = r$ ; and Player 2 randomizes according to the following cumulative distribution function*

$$H_2(x) = \frac{V - (2L_1(r) - L_1(t))}{V} + \frac{1}{V} L_1(x), \quad (12)$$

*on the interval  $[0, t]$ , placing an atom at zero and at  $x = r$ . Where*

$$t = L_2^{-1}(2L_2(r) - V) \quad (13)$$

Consider the following example

**Example 1.** Suppose that  $V = 1$ ,  $L_1(x) = \frac{1}{2}x$ , and  $L_2(x) = x$ .

Then,  $t_2 = 1 < 2 = t_1$  and conditions (1) – (5) hold. From (11) and (12), in the conventional mixed-strategy NE, Player 1 randomizes according to the following cumulative distribution function:

$$H_1(x) = x,$$

on the interval  $[0, 1]$ . Player 2 randomizes according to the following cumulative distribution function

$$H_2(x) = \left(1 - \frac{1}{2}\right) + \frac{1}{2}x,$$

on the interval  $[0, 1]$ , placing an atom of size  $\frac{1}{2}$  at zero.

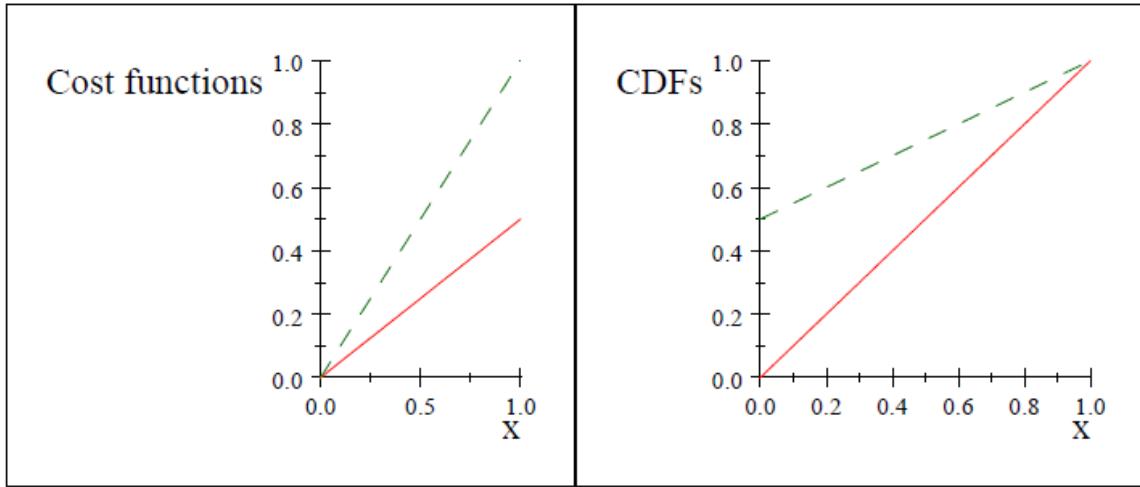


Figure 3.1 Cost Functions and Cumulative Distribution Functions

$L_1(x)$  – solid line and  $L_2(x)$  – dash line on the left panel

$H_1(x)$  – solid line and  $H_2(x)$  – dash line on the right panel

Note that

$$r_0 = \min \left\{ L_1^{-1} \left( \frac{V}{2} \right), L_2^{-1} \left( \frac{V}{2} \right) \right\},$$

$$r_0 = \left\{ 1, \frac{1}{2} \right\} = \frac{1}{2}.$$

So, if the rigid cap  $r \leq r_0$ , then Theorem 1 describes a pure strategy NE.

Consider

$$r = \frac{3}{4}.$$

Theorem 2 describes a mixed strategy NE, where both player will put an atom at  $r = \frac{3}{4}$ ,

but the size of these atoms have to be less than one.

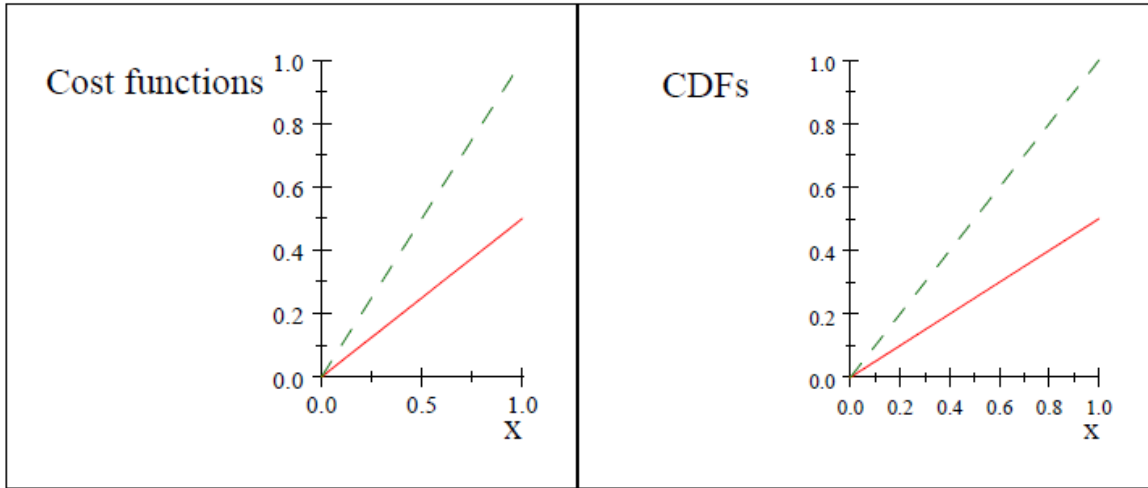


Figure 3.2 Cost Functions and Cumulative Distribution Functions

$L_1(x)$  – solid line and  $L_2(x)$  – dash line on the left panel

$H_1(x)$  – solid line and  $H_2(x)$  – dash line on the right panel

From condition (13), we get

$$t = L_2^{-1} \left( 2L_2 \left( \frac{3}{4} \right) - 1 \right) = \frac{1}{2}$$

Therefore, there exists a mixed-strategy NE, where Player 1 randomizes according to the following cumulative distribution function

$$H_1(x) = x,$$

on the interval  $\left[0, \frac{1}{2}\right]$  placing an atom at  $x = r = \frac{3}{4}$ ; and Player 2 randomizes according to the following cumulative distribution function

$$H_2(x) = \frac{1}{2} + \frac{1}{2}x,$$

on the interval  $\left[0, \frac{1}{2}\right]$ , placing an atom of size  $\frac{1}{2}$  at zero and at  $x = r = \frac{3}{4}$ .

### 3.3 Aggregate Bids and Total Cost

We can now determine the total cost in the asymmetric two-player contest with a rigid cap

$$RTC(r) = \int_0^r L_1(x) dH_1(x) + \int_0^r L_2(x) dH_2(x),$$

and the expected aggregate bids

$$RAB(r) = \int_0^r x dH_1(x) + \int_0^r x dH_2(x).$$

For both the expressions above, the first term gives the total cost (or expected aggregate bids) for Player 1, and the second term gives the same for Player 2.

#### 3.3.1 Small Rigid Cap

Suppose that condition (9) holds, then

$$RTC(r) = L_1(r) + L_2(r),$$

and

$$RAB(r) = 2r.$$



The total cost and the expected aggregate bids are maximized at  $r = r_0$ , or

$$\max_{0 \leq r \leq r_0} RTC(r) = L_1(r_0) + L_2(r_0),$$

and

$$\max_{0 \leq r \leq r_0} RAB(r) = 2r_0.$$

### 3.3.2 Large Rigid Cap

Suppose that condition (10) holds, then we get the following result.

**Theorem 3** *The total costs are*

$$RTC(r) = \begin{cases} L_1(r) + L_2(r), & \text{if } 0 < r \leq r_0 < t_2, \\ 2L_1(r) - L_1(t), & \text{if } r_0 < r < t_2, \end{cases} \quad (14)$$

*and the aggregate bids are*

$$RAB(r) = \begin{cases} 2r, & \text{if } 0 < r \leq r_0 < t_2, \\ \frac{1}{V} \left( t(L_1(t) + L_2(t)) - \int_0^t (L_1(x) + L_2(x)) dx \right) \\ \quad + r(2 - H_1(t) - H_2(t)), & \text{if } r_0 < r < t_2 \end{cases} \quad (15)$$

where  $t$  is defined in (13).

The following examples illustrates.

**Example 2.** Suppose that  $V = 10$ ,  $L_1(x) = x \leq 2x = L_2(x)$ .

Then

$$t_2 = 5 < 10 = t_1,$$

$$r_0 = L_2^{-1}\left(\frac{V}{2}\right) = 2.5,$$

and

$$t = L_2^{-1}(2L_2(r) - V) = 2r - 5.$$

The total cost is then given as

$$RTC(r) = \begin{cases} 3r, & \text{if } 0 < r \leq 2.5, \\ 5, & \text{if } 2.5 < r < 5. \end{cases}$$

and the expected aggregate bids are

$$RAB(r) = \begin{cases} 2r, & \text{if } 0 < r \leq 2.5, \\ 3.75, & \text{if } 2.5 < r < 5. \end{cases}$$

Furthermore, following on from the analysis conducted in chapter 1, for a situation where there is no cap on bids, the total cost is given as

$$TC = C_1(t_2) = 5,$$

and the expected aggregate bids are

$$\begin{aligned} AB &= \frac{1}{V} \left( \int_0^{t_2} x dL_2(x) + \int_0^{t_2} x dL_1(x) \right) \\ &= \frac{1}{V} \left( t_2(L_1(t_2) + L_2(t_2)) - \left( \int_0^{t_2} L_1(x) dx + \int_0^{t_2} L_2(x) dx \right) \right) \end{aligned}$$

$$AB = \frac{1}{10} \left( 5(5 + 10) - \left( \frac{5^2}{2} + 2 \frac{5^2}{2} \right) \right) = 3.75.$$

Note that the total cost with no cap on bids, is the same as that with a large rigid cap, and less than the expected revenue with a small rigid cap in place. Additionally, for a small rigid cap, the total cost and the expected aggregate bids are maximized at  $r = r_0$ , or

$$\max RTC = L_1(r_0) + L_2(r_0) = 7.5,$$

and

$$\max RAB = 2r_0 = 5.$$

This shows that both, the total cost, and expected aggregate bids are maximized at the level of the small rigid cap when compared with a large rigid cap on bids, and when there are no cap on bids.

The figure below plots the expected revenue as a function of a cap on bids when both players face linear cost functions. This plot is in agreement with Che and Gale (1998) who obtain a similar plot for their formulation of cap on bids.

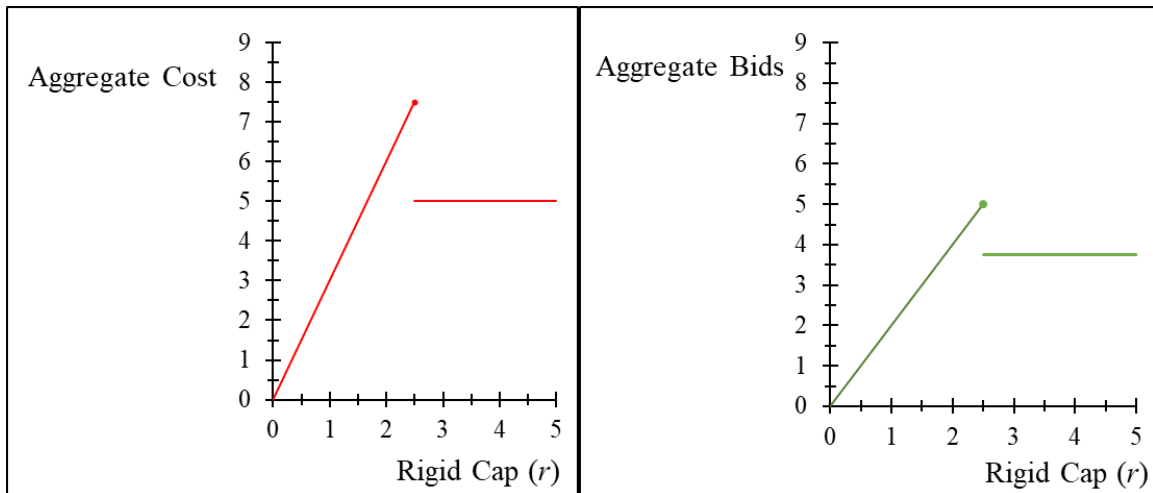


Figure 3.3 Aggregate Cost and Aggregate Bids for a Rigid Cap on bids  
Aggregate Cost vs. size of Rigid Cap ( $r$ ) – on the left panel  
Aggregate Bids vs. size of Rigid Cap ( $r$ ) – on the right panel

#### 4. Conclusions

Using our model, we have also explored the various policy options around the imposition of an exogenous cap on bids, which organizers face when conducting such contests. We consider the impact such policy has on the total cost to the players and aggregate bids made by them. We show that, in terms of increasing the total cost and expected aggregate bids made by players, a policy of small rigid cap on bids strictly dominates over one with no cap on bids. Our findings support Che and Gale (1998) show an increase in total cost for a soft cap with non-monetary penalties. These findings are summarized in Table. 3.1.

Considering various policy options with regards to a rigid cap on bids, a large rigid cap on bids, may dissuade competition by allowing greater leeway to the more efficient player, who could outbid the less efficient player at larger amounts, thereby making the less efficient player not only to be less willing to bid large amounts, but also be more inclined to not participate. Knowing this, the more efficient player would likely bid amounts lower than otherwise. On the other hand, a small rigid cap may have the effect of making competition more even for both players to bid, resulting in an increase in total cost and expected aggregate bids.

As has been discussed earlier in the introduction, total cost and aggregate bids may be of interest to both organizers and player. An organizer would be indifferent between a large cap on bids and no cap on bids from the point of view total cost and aggregate bids. A small rigid cap is more likely to generate higher total cost and higher expected aggregate bids.

**Table 3.1 Summary of results pertaining to a rigid cap on bids**

This table summarizes our findings with regards to rigid cap on bids, and compares it with previous literature on this subject.

	Che and Gale (1998)	Kaplan and Wettstein (2006)	Che and Gale (2006)	Olszewski and Siegel (2019)	This paper
Aggregate Costs					
Rigid Cap	--	--	Increases	Decreases	Increases for small rigid cap
No Cap	--	Dominates	--	--	Same as large rigid cap
Aggregate Bids					
Rigid Cap	Increase	--	--	Decreases	Increases for small rigid cap
No Cap	--	Dominates	--	--	Same as large rigid cap

## CHAPTER 4

### INTERNAL MIGRATION AND URBANIZATION IN INDIA

#### **1. Introduction**

Since the end of the Second World War, the world population has been urbanizing at a continuous rate. According to the World Urbanization Prospects 2014 report commissioned by the UN, while 30% of the world population in 1950 was urbanized, this figure stood at 54% in 2014, and is projected that by 2050, close to two-thirds of the global population will be residing in urban settings. Given the current levels of urbanization in the rest of the world, much of the growth in urbanization would come from Africa and Asia. The UN in its report expects about 37% of this projected growth to come from just three countries viz. India, China, and Nigeria. Discussions focused around this trend of increasing urbanization eventually bring into picture, the enchanting notion of megacities around the world, especially in Asia with cities like Tokyo, Shanghai, Beijing, Guangzhou, Mumbai, Delhi, Dhaka etc. capturing imaginations with their already massive populations, and predictions of unprecedentedly large conurbations. However, as noted by the UN in the aforementioned report, the fastest growing urban agglomerations are not the mega, but the medium-sized cities.

#### *Urbanization in India in the 20<sup>th</sup> Century*

The close relationship between urbanization and economic development has long been understood. As the economy of a country grows, cities develop as centers of trade and

commerce, and allied services. India is no exception to that phenomenon. Urban population in India has grown since at least the late 19<sup>th</sup> century when regular population census was instituted under the British Raj. The British were instrumental in the building and establishment of cities like Kolkata (Calcutta), Mumbai (Bombay), and Chennai (Madras) as port cities open to international trade. These port cities attracted manual labor for loading and unloading the ships, for handling the cargo, and the like. These cities also attracted various business communities from across the country, and the literate others who could work as accountants, managers, supervisors at the docks, warehouses (godowns) etc. The British also played an important role in increasing the prominence of existing older cities like Bengaluru (Bangalore) and Pune, which owing to their relatively pleasant climate and relative proximity to Bombay in the case of Pune, and to Madras in the case of Bangalore, made for substantial British settlements, leading eventually to the establishment of some prominent military, medical, educational and other institutes in Bengaluru and Pune. The British rule, unlike what many might imagine, was not uniform across the entire Indian subcontinent. Although there were large parts of what is now India, directly controlled and ruled over by the British, there were equally large parts that were not under direct British rule, but were controlled indirectly by subjugation and acquiescence of local rulers or ‘princes’. While the local princes had nominal powers, the cities under their rule did not “enjoy” the same facilities as those that were directly ruled by the British. An example of this is the establishment of ‘railway towns’, which almost exclusively were located in the territory ruled over by the British directly. The partition of India in 1947 (essentially the partition of the north-western region of Punjab and the eastern region of Bengal) saw the population of many cities in what became northern, western, eastern, and central India

increase sharply owing to the influx of refugees from what became Pakistan. The population of Delhi, for example, almost doubled in the census period of 1941 – 1951, from being under a million in 1941 to being under two million in 1951. This study will not be focusing on this episodic spike in urban population. Given this historical background to the process of urbanization in India, we now proceed to talk about processes closer to the present time.

In the period after 1947 with India gaining independence, cities mentioned above and a few others, therefore had a head start of sorts, relative to cities that were not under direct British rule. These cities had better infrastructure, higher quality of human capital (both, educationally, and entrepreneurially) and no surprise then that many of these cities are among the largest cities in India today.

### *Linguistic Diversity in India*

A development, which may seem unrelated to the discussion on urbanization is the laying down of India's federal structure of administrative units. The drafters of the Constitution of India were keen to take what to them, was best in the American Constitution among others such documents from around the world, and what they had been familiar with under the British Parliamentary system. This led to the creation of States in India as the second level of administration, much like in the United States. However, unlike in the United States, there are multiple languages spoken in India, many with their own scripts and grammar, which distinguishes India even from much of Europe where for example, English, French, German, and Italian are all written in the Latin script with relatively minor differences in the alphabet. A person from England, for example, can read most of what might be written in Italian, they may not know the meaning of the words or how to



pronounce the words correctly. In India however, there are sixty languages with numerous scripts that are spoken by at least 100,000 people and many other languages spoken by smaller populations. In such a situation, there was a popular demand for language to be the criteria for state formation, to which the government of the day reluctantly acceded. The States Reorganisation Commission was created in 1953 to reorganize States on linguistic basis. Andhra Pradesh was the first state to be created on this basis in 1956 with the majority of its population speaking the state language – Telugu. Numerous states created on linguistic basis followed Andhra Pradesh. The constitution of India currently recognizes 26 languages as official languages, while specifying that India has no national language. Hindi, the language spoken by the largest share of the population, and English are to be used by the Central (Federal, as per US terminology) Government and the Supreme Court of India for official proceedings. An Indian bank note for example, carries on it the value written in 17 languages in 12 different scripts including Hindi and English as the most prominent ones. This study argues that the linguistic diversity, specifically the linguistic nature of Indian states is an important factor to be considered in the urbanization process in India. Unlike other large countries like the US, China, or Brazil for example, India does not have a single language that can be designated as the lingua franca. It is therefore understandable why this aspect of India could, and does get overlooked, and so it seems worthwhile to look into the evolving urbanization patterns in India in that context and pay closer attention to such factors that on surface perhaps, even seem immaterial.

## **2. Literature Review**

In the most recent study relevant to this inquiry, Chauvin et al. (2017) look into urbanization across four countries viz. Brazil, China, India, and the United States. The

study concludes that both China and India have fewer extremely large cities than would be predicted by Zipf's law. Chauvin et al. also show that despite increasing urbanization in the country, labor mobility in India is very low. They find the low rates of migration within India as "puzzling" given the rapid growth of cities. Munshi and Rosenzweig (2009) also find the mobility rate to be low in India and attribute that to the informal risk-sharing within caste networks, wherein, if an individual were to migrate, and move away from their caste network, they would lose the insurance or security provided by their local caste network. The discussion in Morten (2013) on the other hand, suggests that low mobility numbers for India are likely due to measurement errors and that the Indian data understates the actual numbers. This would address the paradox of fast growth of cities despite recorded low mobility. Also related to the issue of measurement errors and understating of actual numbers, Tumbe (2016) draws attention to the role played by definitions. According to him, India uses a rather conservative approach when it comes to defining and classifying of what is 'urban' and what is 'rural'. As a result, many towns and villages in India that would be classified as 'urban' in other countries, get classified as 'rural' in India. A more liberal definition in keeping with practices in much of the world would see the urbanization rate in India increase from 31% in 2011 to 47%. Keeping in mind the fast growth of cities in India, Jedwab and Vollrath (2016) also draw attention to high fertility rates, which would also explain fast urban growth regardless of mobility. This paper seeks to add another dimension, namely linguistic diversity, to the discussion around mobility, or the lack thereof, in India. As has been discussed above, given the linguistically organized states in India, immigration would be hampered across state borders. An individual would be less likely to go to another state where the language is different from the one spoken within his

or her native state. On the other hand, migration within a state would be much more significant than migration across states.

In addition to highlighting the puzzling nature of mobility in India, Chauvin et al. (2017) also show that there is a very high wage premium for urban areas in India compared to the United States, Brazil, and China. The finding that there exists a very high urban wage premium, raises further questions. A simple question is that if urban wage premium is very high, then the rate of urbanization should at least not be as low as it is in India. Due the absence of any verifiable explanation, the authors suggest that the high urban wage premium indicates much higher human capital levels, especially educational attainment, in urban Indians than their rural compatriots. Another plausible explanation according to them could be down to poor data. This paper explores the possibility of another language-based explanation, this time to do with proficiency in English, to look into the high urban wage premium. We argue that English-language skills can be considered as a form of human capital, the possession of which allows for a better paying job. It is widely acknowledged from anecdotal evidence that English language skills are considered valuable in India. Studies like Roy (2004) finds that as a result of the change in medium of instruction in West Bengal public primary schools from English to Bengali in 1983, parents spent more on private tutors, presumably to provide English lessons. This shows the value attached to English language education. Munshi and Rosenzweig (2006) and Chakraborty and Kapur (2016), both estimate the returns to attending a school with English (as opposed to some Indian language) as the medium of instruction. Munshi and Rosenzweig collected their own data on Marathi-speakers living in Dadar, which is located in Mumbai, Maharashtra, India. Using data on parents' income histories and the language of instruction in their

secondary school (Marathi or English), they estimate significant positive returns to an English-medium education. Attending an English-medium school increased both women's and men's income by about 25% in 2000. Chakraborty and Bakshi (2016) use National Sample Survey data to estimate the impact of a 1983 policy in West Bengal which eliminated English as the medium of instruction in primary schools. They find that switching from English to Bengali medium of instruction significantly reduced wages. Simple comparisons of cohorts attending primary school before and after the policy change suggest that English-medium schooling raised wages about 15% in the 2000s. While these studies highlight the importance and value of English language education, this study is more closely related to those done by Shastry (2012) who finds that Indian districts whose population's mother tongue is more linguistically dissimilar to Hindi attract more information technology (IT) jobs, which she attributes to the lower cost of learning English<sup>12</sup>. Then she finds that a greater IT presence is associated with a greater increase in school enrollment and a smaller increase in the wage premium for educated workers in districts where the mother tongue is more linguistically dissimilar to Hindi. However, she does not have individual-level data on English-language skills her language variables are at the district level and does not estimate returns to English proficiency per se. Clingingsmith (2014) finds that districts that had greater increases in manufacturing employment experienced greater increases in the proportion of minority-language speakers becoming bilingual (where the second language is a regional or national language). Azam et. al (2010) study the returns to English-language skills in India and find large difference in earnings between those who do, and those who do not speak English. These results shed

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<sup>12</sup> The reasoning is that people whose native language is not Hindi or English will learn Hindi if their mother tongue is very similar to Hindi and English otherwise.

an important light on the processes of urbanization in India in absolute, and in comparative terms. However, Chauvin et al. (2017) are unable to provide any substantial explanation for these finding.

### **3. Research Question and Hypothesis**

Some of the questions that we seek to address are those raised by Chauvin et al. (2016) in that we wish to examine if linguistic diversity is a factor in labor mobility in the Indian context. We contend that linguistic diversity in India does play a significant role in determining the movement of people across India as it urbanizes, and that labor, especially, poorer and low skilled, would be restricted in their mobility by linguistic boundaries. Another question raised by Chauvin et al. (2017) is regarding very high urban wage premium in India. We seek to explore a linguistic explanation for this phenomenon as well, specifically, is there any correlation between the high wage premiums in urban areas to English being predominantly an urban language? The argument here is that while poorer, low-skilled workers might be limited in their mobility due to the linguistic diversity, the more educated, middle, and lower-middle class workers can transcend such linguistic boundaries owing to their better knowledge of the English language, and that such workers drive up the urban wages.

### **4. Data**

We use data from the 2011-12 India Human Development Survey (IHDS-II), a nationally representative household data set collected by the National Council of Applied Economic Research in New Delhi and the University of Maryland (Desai, Reeve and NCAER 2011). IHDS-II covers 204,569 individuals located throughout India. One of the

most relevant for us is that information pertaining to languages spoken, and read, and especially each household member's ability to converse in English is collected. We are not aware of any other large-scale individual-level data set in India that contains a measure of English-language skills. Aggregated data from the census survey of India has been employed to support the analysis carried out using the IHDS-II dataset. Data pertaining to languages across India has been sourced from the 2011 census survey of India. The 2011 census survey is the most recent source for migration related aggregated data. Since the outcome of interest is earnings, we restrict our sample to individuals aged 18 to 65.

In the following sections, we address the questions raised in section 4, viz., is there a linguistic explanation for low mobility across India? And, does English language proficiency explain high urban wage premium?

## **5. Mobility across State Borders in India**

As per Chauvin et al. (2017), labor mobility in India is around 2%, the IHDS-II dataset also reflects this, as has been shown in Table. 4.1. In this section, we will be looking for a linguistic explanation for the low mobility in India. Specifically, is linguistic diversity in India, a hindrance to mobility? The argument here is quite straightforward, in that, one is less likely to migrate to a place(s) where the language spoken is different. In the Indian context, where states are organized on a linguistic basis, the most widely spoken language is Hindi (mother tongue in at least 10 states, and second language in some other states). There are nonetheless many states with their own languages and scripts. Arguably, the knowledge of English would allow an individual to transcend these linguistic barriers. The IHDS-II dataset asks the respondents if they had left their homes in the last five years and gone off to another state to look for temporary employment. In case the response is a yes,

the next question pertains to the duration of time the respondent had been away. We carry out a logistical regression approximated by the following equation:

$$Y_i = \alpha + \beta English_i + \rho X_i + \varepsilon_i \quad (1)$$

where  $Y_i$  is the binary response to the question whether individual  $i$  temporarily migrated out-of-state in the last five years or not.  $English_i$  is a binary dummy for English speaking ability which is self-reported by individual  $i$ ,  $X_i$  are controls such as income, age, and highest level of education.  $\beta$  is the coefficient under consideration; it gives the log-likelihood of a ‘Yes’ response to the question, has an individual temporarily migrated out-of-state in the last five years given that they possess English speaking abilities? The individual respondents here are from both, urban and rural areas, and are also questioned regarding the duration of time for which they temporarily migrated. These results are shown in Table 4.2, with columns 1 through 3 show the log-likelihood of a temporary migration across state borders from both rural and urban areas. Column 1 shows the results in the case that the temporary migration was for a duration of less than 1 year; column 2, when it was more than 1 year; and column 3, when the duration of the temporary migration across state borders is for more than 2 years. For the results in these columns, we control for income, highest level of education, and age, in addition to the English-speaking skills for a male who is older than 24 years of age. Notice that while the coefficients for English speaking ability are positive, they are not significant. Furthermore, columns 4, 5, and 6, show the results of the logistic regression after dropping the highest level of education as one of the controls. The coefficient for English speaking ability become even less significant in this case. The log odds of temporarily migrating out-of-state in the last five years for a duration of time more than 1 year, and more than 2 years increases for men

above 24 years of age who can speak English. Coefficients for other explanatory variables are insignificant. We can therefore conclude that while English speaking ability increases the likelihood that an individual would temporarily migrate across state borders, it however is not significant, and even less significant if one does not take into consideration the highest level of education for the individuals. In addition to the logistic regression, we also perform a Poisson regression for equation (1) and obtain the incident rate ratios, which are contained in Table 4.3. The results show that the incident rate for English speakers is 1.1 times the incident rate for non-English speakers when it comes to migrating temporarily across state borders, which is not significantly large.

We now consider Hindi language as a factor in movement across state borders. As has been mentioned above, Hindi is the mother tongue of at least ten out of the 29 states, and is a second language in some other states. That said, there are many states where Hindi is neither the state-language, nor is it spoken or understood in any significant measure. We therefore use as control, a binary variable for Hindi *not* as the official state language. The binary variable takes the value 1, if Hindi is not the official state language, and zero otherwise. A logistic regression is performed on the following equation (2):

$$Y_i = \alpha + \beta_1 English_i + \beta_2 Hindi\_official_i + \rho X_i + \varepsilon_i \quad (2)$$

where  $Y_i$  is the binary response to the question whether individual  $i$  temporarily migrated out-of-state in the last five years or not.  $English_i$  is a dummy for self-reported English speaking ability as was the case with equation (1),  $Hindi\_official_i$  is the dummy for whether the official language of a state  $i$  is *not* Hindi.  $X_i$  are controls such as income, age, residence (urban or rural) and highest level of education. Results of the logistic regression for equation (2) are presented in Table 4.4, where one can notice that Hindi not being the



official language in a state is significant hurdle to the movement across state borders. The coefficient for ‘Hindi not official state language’, is negative and highly significant for all durations of temporary migrations as shown in columns (1), (2), and (3), of Table 4.4. Moreover, upon controlling for Hindi as the official language at the state level, English speaking ability is positive and significant at 5% level. The size of the coefficient rises slightly with the duration, even as the coefficient for English speaking ability is insignificant when the duration of temporary migration across state borders is less than 1 year. The result from Table 4.4 shows that for states where the official language (mother tongue) is not Hindi, most migrants would come from within the state, and not outside the state. This inference is corroborated by the 2011 census data for migrants across various cities in India. Table 4.5 contains this information extracted from 2011 census survey of India. This table does not include Delhi among the cities listed; this is because Delhi is akin to a ‘city-state’, similar to the District of Columbia in the United States, where by its very nature, most migrants come from outside the city. It is nonetheless, quite telling, that even IT-Hubs like, Bangalore and Hyderabad have 56% and 81% of their long-term immigrant residents respectively, from within the state instead of them belonging from other parts of the country.

## **6. Human Capital and Migration in India**

Chauvin et al. (2017) find substantial wage premium associated with Indian cities as compared to cities in the US, Brazil, and China. They are however, unable to explain why this might be the case. They speculate that this high wage premium in Indian cities might point to some unobserved human capital. In this section we will attempt to explore any relationship between human capital and urbanization in the Indian context. Such a

connection, we expect, will shed some light on the high wage premiums associated with cities in India. The relationship between migrating to a city and human capital might be approximated by the following equation (3):

$$Y_i = \alpha + \beta_1 \text{English}_i + \beta_2 \text{Highest level of education}_i + \rho X_i + \varepsilon_i \quad (3)$$

where  $Y_i$  is the binary choice variable of whether an individual  $i$  migrated on a temporary basis to an urban or a rural destination in the last five years.  $\text{English}_i$  is a dummy for English speaking ability of an individual  $i$  which is self-reported.,  $\text{Highest level of education}_i$  for an individual  $i$  is a measure of human capital that the individual possesses.  $X_i$  are controls such as income, age, residence (urban or rural). The observations are restricted to males older than 24 years of age. Table 4.6 contains the results of the logistic regression performed on equation (3). The results show that the coefficient for  $\text{Highest level of education}_i$  are strongly significant and positive. More educated men who are more than 24 years old, are more likely to migrate temporarily to an urban area for a duration of up to 1 year, and more than 1 year. Coefficient for the duration in excess of 2 years is positive, but less significant. This might be due to small number of observations. Furthermore, for those who reside in rural areas as well, better educated men aged 24 years and above, are more likely to migrate temporarily to an urban area. As more educated people move to the cities, this perhaps explains high wage premiums in Indian cities. That said, cities in other parts of the world also attract more educated individuals with specialist jobs opportunities. In the Indian context then, either more educated individuals are a lot more scarce compared to other countries, or that there is a much greater concentration of jobs requiring higher education in Indian cities compared to cities in other countries. This requires further exploration, which is beyond this scope of this study. It is

quite possible that both scenarios coexist. It is well known that for the most part of the last 70 years since India's independence, the country has suffered from shoddy infrastructure across the country of the most basic kind. This has over the years meant that many of the jobs that do require a workforce that is well endowed in human capital, are concentrated in a handful of cities where the infrastructure is relatively much better developed. Furthermore, as has been shown in the previous section, mobility across state borders is hindered by linguistic diversity. This would imply that well-educated individuals may not migrate to different parts of the country in their job seeking efforts, and hence, there might be a scarcity of well-educated people in Indian cities, causing the wage for such individuals to attract a significant premium.

## **7. Conclusion**

We have tried to address some of the issues that had been raised by Chauvin et al. (2016). In doing so, we have tried to consider linguistic diversity in India, and explore aspects of human capital and its relationship with migration. We show that linguistic diversity does affect the mobility of individuals across state borders in India. Specifically, if Hindi is not the official language of a state, the individual residents find it quite a barrier to overcome to be able to go to different parts of the country. Controlling for Hindi not being a state's official language, we find that the self-reported English speaking ability positively affects mobility across state borders.

We also show that cities in India do attract the more highly educated individuals including those from rural areas. This may be due to urban-rural disparity in terms of the types of jobs available in each of these places. Certain jobs might almost be monopolized in the urban areas, e.g. good university/academic jobs, which are almost exclusively in

urban settings in India; and same is the case with hospitals, manufacturing, and IT related industry. Cities in India have had the advantage over rural areas in terms of better basic infrastructure, which has allowed for almost all of well-paying jobs that require higher education, to be almost exclusively based out of cities. Not to mention that if even the well-educated individuals find it difficult to be mobile across state borders, then that may create a shortage of well-educated workers, pushing the wages up, and therefore we find substantial wage premium associated with Indian cities as compared cities in other parts of the world. Finally, we would also like to mention some shortcomings of our study that are borne out of the IHDS-II dataset. The dataset considers and reports on only those individuals who migrated temporarily, and returned back to their own homes in the last five years from the date of the interview. A more elaborate study would be able to include migrants to Indian cities who have decided to stay and not return to their native towns and villages. Currently the IHDS-II dataset does not allow for such an exploration. Another issue with the dataset is that the number of observation reduces drastically as one tries to include variables such as the self-reporting English speaking ability. Small numbers of observations might lead to untrustworthy findings.

**Table 4.1 Temporary Migrant in the last five years**

This table presents a summary of the number of individual respondents to the IHDS-II survey who had migrated out of their place of residence temporarily in the last five years, and have since returned.

	Frequency	Percentage	Cumulative
No	201,280	98.39	98.39
Yes	3,289	1.61	100.00
Total	204, 569	100.00	

**Table 4.2: Temporary Out-of-State Migration**

This table shows the impact of various variables on the choice and the likelihood of an individual to migrate outside their own state.

	(1)	(2)	(3)	(4)	(5)	(6)
	Duration less than 1yr.	Duration more than 1yr.	Duration more than 2yrs.	Duration less than 1yr.	Duration more than 1yr.	Duration more than 2yrs.
English	0.205 (0.133)	0.504* (0.302)	0.736* (0.382)	0.175 (0.120)	0.358 (0.260)	0.021 (0.411)
Highest level of education	-0.006 (0.011)	-0.025 (0.026)	-0.052 (0.035)			
Log of income	-0.078* (0.047)	-0.063 (0.110)	0.127 (0.153)	-0.084* (0.046)	-0.083 (0.108)	0.100 (0.161)
Age	-0.012** (0.005)	0.005 (0.011)	0.008 (0.015)	-0.011** (0.004)	0.007 (0.011)	0.015 (0.019)
constant	1.470*** (0.520)	1.052 (1.280)	-1.073 (1.790)	1.486*** (0.519)	1.049 (1.285)	-1.299 (2.010)
Obs.	2102	403	226	2102	403	148

*Note:* observations include males older than 24 yrs. of age, from both, rural and urban areas.

Standard errors are in parenthesis.

Asterisks denote significance levels (\* = 0.1, \*\* = 0.05, \*\*\* = 0.01)

**Table 4.3: Temporary Out-of-State Migration Poisson Regression**

This table shows results for a Poisson regression where each column reports the incident rate ratio (IRR).

	(1)	(2)	(3)
	Duration less than 1yr. (IRR)	Duration more than 1yr. (IRR)	Duration more than 2yrs. (IRR)
English	1.093 (0.095)	1.204* (0.216)	1.335* (0.313)
Highest level of education	0.997 (0.007)	0.990 (0.016)	0.979 (0.022)
Log of income	0.966* (0.030)	0.977* (0.065)	1.052 (0.100)
Age	0.995** (0.003)	1.002 (0.006)	1.003 (0.009)
constant	0.968*** (0.330)	0.762 (0.590)	0.330 (0.371)
Obs.	2102	403	226

*Note:* observations include males older than 24 years of age from both, rural and urban areas.

Standard errors in parenthesis

Asterisks denote significance levels (\* = 0.1, \*\* = 0.05, \*\*\* = 0.01)

**Table 4.4: Temporary out-of-state migration when Hindi not official state language**

This table presents the result of a logistic regression carried out to ascertain the likelihood that a man who is older than 24 years of age would migrate out-of-state. The table includes the variable for Hindi to not be the official language in the state.

	(1) Duration less than 1 yr.	(2) Duration more than 1 yr.	(3) Duration more than 2 yrs.
english	0.186 (0.140)	0.631** (0.314)	0.927** (0.402)
Highest level of education	0.010 (0.012)	-0.004 (0.027)	-0.037 (0.036)
Log of income	0.013 (0.050)	0.023 (0.117)	0.215 (0.166)
Urban residence	0.244 (0.151)	0.072 (0.347)	-0.228 (0.415)
Hindi not official state language	-1.308*** (0.096)	-1.286*** (0.228)	-1.266*** (0.304)
Age	-0.007 (0.005)	0.011 (0.012)	0.015 (0.016)
constant	0.776 (0.550)	0.289 (1.359)	-1.741 (1.927)
Obs.	2102	403	226

*Note:* observations include males older than 24 years of age from both, rural and urban areas.

Standard errors are in parenthesis

Asterisks denote significance levels (\* = 0.1, \*\* = 0.05, \*\*\* = 0.01)



**Table 4.5 Share of Total Migrants from within India for selected large cities.**

This table presents a snapshot of the share of total immigrants, both recent, and long-term, who migrate to a large city from either within the same state, or from outside the state. The table below also highlights the different official languages in different states, alongside the second languages.

City	State	Official State Language <i>Second Language</i>	Duration of Residence	Share of Total (Urban and Rural) Migrants from within India	
				From within the State	From outside the State
Mumbai (Bombay)	Maharashtra	Marathi	1 - 4 years	35%	65%
		Hindi	10 yrs. and more	43.70%	56.30%
Kolkata (Calcutta)	West Bengal	Bengali	1 - 4 years	63.40%	36.60%
		Hindi	10 yrs. and more	52.70%	47.30%
Chennai (Madras)	Tamil Nadu	Tamil	1 - 4 years	76.90%	23.10%
		Telugu	10 yrs. and more	78.80%	21.20%
Bengaluru (Bangalore)	Karnataka	Kannada	1 - 4 years	51.50%	48.50%
		Tamil	10 yrs. and more	56.10%	43.90%
Hyderabad	Andhra Pradesh	Telugu	1 - 4 years	81.50%	18.50%
		Urdu	10 yrs. and more	81.50%	18.50%
Ahmedabad	Gujarat	Gujarati	1 - 4 years	68.50%	31.50%
		Hindi	10 yrs. and more	69.20%	30.80%
Pune	Maharashtra	Marathi	1 - 4 years	63.60%	36.40%
		Hindi	10 yrs. and more	75.40%	24.60%
Surat	Gujarat	Gujarati	1 - 4 years	49.40%	50.60%
		Hindi	10 yrs. and more	54.50%	45.50%
Jaipur	Rajasthan	Hindi	1 - 4 years	63.00%	37%
		Urdu	10 yrs. and more	69.80%	30.20%

*Source:* Census of India 2001, Table D-3

*Notes:* Delhi is not included in the list because it is “city state”, and all immigrants into Delhi are by definition from outside Delhi and the distinction as made in the last two columns, cannot be made for Delhi. The more recent, 2011 census could not be used as this specific data has not yet been made publicly available.

**Table 4.6: Temporary Migration to Urban Area**

This table shows the influence of a variety of factors on an individual's choice to temporarily migrate to an urban area in India.

	(1) Duration less than 1yr.	(2) Duration more than 1 yr.	(3) Duration more than 2 yrs.
english	-0.046 (0.150)	0.251 (0.340)	0.130 (0.438)
Highest level of education	0.070*** (0.012)	0.088*** (0.029)	0.056 (0.038)
Log of income	-0.093* (0.052)	-0.178 (0.125)	0.070 (0.173)
Rural residence	-0.624*** (0.173)	-0.657 (0.402)	-0.375 (0.472)
Hindi not official state language	-0.244** (0.098)	-0.378 (0.232)	0.089 (0.306)
Age	-0.015*** (0.005)	0.001 (0.012)	-0.013 (0.016)
constant	2.519*** (0.614)	2.842* (1.553)	0.458 (2.147)
Obs.	2101	403	226

*Note:* observations include males older than 24 years of age from both, rural and urban areas.

Standard errors are in parenthesis

Asterisks denote significance levels (\* = 0.1, \*\* = 0.05, \*\*\* = 0.01)

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## APPENDIX A

### PROOF FOR THEOREMS IN CHAPTER 2

**Proof of Theorem 1.** First, since conditions (1) – (6) hold, functions  $F_1$  and  $F_2$  in (8) and (9) are strictly increasing, right-continuous, and indeed cumulative distribution functions<sup>13</sup>. Second, it is straightforward to check that if Player 1 randomizes according to the cumulative distribution function  $F_1(x)$  on the interval  $[0, t_2]$ , then Player 2's expected payoff for bidding  $x \in [0, t_2]$  is

$$VF_1(x) - C_2(x) = V \frac{1}{V} C_2(x) - C_2(x) = 0.$$

Analogously, if Player 2 randomizes according to the cumulative distribution function  $F_2(x)$  on the interval  $[0, t_2]$ , then Player 1's expected payoff for bidding  $x \in [0, t_2]$  is

$$VF_2(x) - C_1(x) = V \left( \frac{V - C_1(t_2)}{V} + \frac{1}{V} C_1(x) \right) - C_1(x) = V - C_1(t_2) > 0.$$

Therefore, a mixed-strategy NE is obtained. ■

**Proof of Theorem 3.** if  $t_2 < s$ , then from (10) and (11) we get

$$\begin{aligned} STC &= \int_0^{t_2} C_1(x) dF_1(x) + \int_0^{t_2} C_2(x) dF_2(x) \\ STC &= \frac{1}{V} C_1(t_2) C_2(t_2) = C_1(t_2). \end{aligned}$$

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<sup>13</sup> For the definition of a cumulative distribution function, see, for example, Olofsson and Andersson (2012).

Suppose that  $t_2 \geq s$ . Then,

$$\begin{aligned}
STC &= \frac{1}{V} \int_0^s L_1(x) dL_2(x) + \frac{1}{V} \int_s^{t_2} (L_1(x) + m) d(L_2(x) + m) + \\
&\quad \frac{1}{V} \int_0^s L_2(x) dL_1(x) + \frac{1}{V} \int_s^{t_2} (L_2(x) + m) d(L_1(x) + m), \\
&= \frac{1}{V} L_2(s) L_1(s) + \frac{1}{V} \int_s^{t_2} (L_1(x) + m) d(L_2(x) + m) + \\
&\quad \frac{1}{V} ((L_2(t_2) + m)(L_1(t_2) + m) - (L_2(s) + m)(L_1(s) + m)) \\
&\quad - \frac{1}{V} \int_s^{t_2} (L_1(x) + m) d(L_2(x) + m), \\
STC &= C_1(t_2) - \frac{m}{V} (L_1(s) + L_2(s) + m).
\end{aligned}$$

Analogously, for aggregate bids with soft caps,

$$\begin{aligned}
SAB &= \int_0^{t_2} x dF_1(x) + \int_0^{t_2} x dF_2(x) \\
SAB &= \frac{1}{V} \left( \int_0^s x dL_2(x) + \int_s^{t_2} x dL_2(x) + \int_0^s x dL_1(x) + \int_s^{t_2} x dL_1(x) \right), \\
&= \frac{1}{V} \left( \left( x(L_2(x) + L_1(x)) \right) \Big|_0^s + \left( x(L_2(x) + L_1(x)) \right) \Big|_s^{t_2} \right. \\
&\quad \left. - \left( \int_0^s L_1(x) dx + \int_s^{t_2} L_1(x) dx + \int_0^s L_2(x) dx + \int_0^{t_2} L_2(x) dx \right) \right), \\
SAB &= \frac{1}{V} \left( t_2 (L_1(t_2) + L_2(t_2)) - \left( \int_0^{t_2} L_1(x) dx + \int_0^{t_2} L_2(x) dx \right) \right).
\end{aligned}$$

■

## APPENDIX B

### PROOF FOR THEOREMS IN CHAPTER 3

**Proof of Theorem 1.** Suppose that conditions (1) – (5) and (9) hold and each player bids the rigid cap  $r$ . Then, player  $i$  gets the following payoff

$$u_i(r, r) = \frac{V}{2} - C_i(r) \geq 0$$

and cannot increase her payoff given the bid of her opponent. ■

**Proof of Theorem 2.** First, since conditions (1) – (5), and (10) hold, functions  $H_1$  and  $H_2$  in (12) and (13) respectively, are strictly increasing and indeed cumulative distribution functions.

Second, it is straightforward to check that if Player 1 randomizes according to the cumulative distribution function  $F_1(x)$  on the interval  $[0, t]$ , then Player 2's expected payoff for bidding  $x \in [0, t]$  is

$$VH_1(x) - L_2(x) = V \frac{1}{V} L_2(x) - L_2(x) = 0.$$

Suppose that Player 2 bids  $x = r$ . Then, her expected payoff is

$$VH_1(t) + \frac{1}{2}(1 - H_1(t))V - L_2(r) = 0,$$

where the first term is the expected benefit, if Player 1 bids in the interval  $[0, t]$ , the second term is the expected benefit, if Player 1 bids exactly the rigid cap,  $r$ , and the last term is



the cost of bidding. The expected payoff has to be the same as the expected payoff after Player 2 bids on the interval  $[0, t]$ .

We get

$$H_1(t) = \frac{2}{V}L_2(r) - 1.$$

Since

$$H_1(t) = \frac{1}{V}L_2(t),$$

We obtain

$$L_2(t) = 2L_2(r) - V,$$

or

$$t = L_2^{-1}(2L_2(r) - V).$$

Analogously, if Player 2 randomizes according to the cumulative distribution function  $H_2(x)$  on the interval  $[0, t]$ , then Player 1's expected payoff for bidding  $x \in [0, t]$  is

$$VH_2(x) - L_1(x) = VH_2(t) - L_1(t).$$

Suppose that player 1 bids  $x = r$ . Then, her expected payoff is

$$VH_2(t) + \frac{1}{2}(1 - H_2(t))V - L_1(r) = VH_2(t) - L_1(t),$$

since she has to be indifferent among her bids.

Therefore,

$$\frac{1}{2}(1 - H_2(t))V = L_1(r) - L_1(t) \quad (\text{i})$$

and

$$\frac{1}{2}(1 + H_2(t))V - VH_2(x) = L_1(r) - L_1(x). \quad (\text{ii})$$

summing (i) and (ii), we get

$$V - VH_2(x) = 2L_1(r) - L_1(x) - L_1(t),$$

or

$$H_2(x) = 1 - \frac{1}{V}(2L_1(r) - L_1(x) - L_1(t)).$$

Therefore, a mixed-strategy NE is obtained. ■

### **Proof of Theorem 3.**

$$\begin{aligned} RTC(r) &= \int_0^r L_1(x) dH_1(x) + \int_0^r L_2(x) dH_2(x) \\ &= \int_0^t L_1(x) dH_1(x) + L_1(r)H_1(r) + \int_0^t L_2(x) dH_2(x) + L_2(r)H_2(r) \end{aligned}$$

or

$$\begin{aligned} &= \frac{1}{V} \int_0^t L_1(x) dH_2(x) + L_1(r)(1 - H_1(t)) + \\ &\int_0^t L_2(x) d\left(\frac{V - (2L_1(r) - L_1(t))}{V} + \frac{1}{V}L_1(x)\right) + L_2(r)(1 - H_2(t)), \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{V} \int_0^t L_1(x) dL_2(x) + L_1(r)(1 - H_1(t)) + \frac{1}{V} \int_0^t L_2(x) dL_1(x) + L_2(r)(1 - H_2(t)), \\
&= \frac{1}{V} L_1(t)L_2(t) + L_1(r)(1 - L_1(t)) + L_2(r)(1 - H_2(t)), \\
&= \frac{1}{V} L_1(t)L_2(t) + L_1(r) \left( 1 - \frac{1}{V} L_2(t) \right) \\
&\quad + L_2(r) \left( 1 - \left( \frac{V - (2L_1(r) - L_1(t))}{V} + \frac{1}{V} L_1(t) \right) \right), \\
&= \frac{1}{V} L_1(t)L_2(t) + L_1(r) \left( 1 - \frac{1}{V} L_2(t) \right) + L_2(r) \left( 1 - \left( \frac{V - 2(L_1(r) - L_1(t))}{V} \right) \right), \\
&= \frac{1}{V} L_1(t)L_2(t) + L_1(r) \left( 1 - \frac{1}{V} L_2(t) \right) + L_2(r) \left( \frac{2(L_1(r) - L_1(t))}{V} \right), \\
&= L_1(r) + \frac{1}{V} \left( L_1(t)L_2(t) - L_1(r)L_2(t) + 2L_2(r)(L_1(r) - L_1(t)) \right),
\end{aligned}$$

since

$$t = L_2^{-1}(2L_2(r) - V),$$

we get

$$\begin{aligned}
RTC(r) &= L_1(r) \\
&\quad + \frac{1}{V} \left( L_1(t)(2L_2(r) - V) - L_1(r)(2L_2(r) - V) \right. \\
&\quad \left. + 2L_2(r)(L_1(r) - L_1(t)) \right),
\end{aligned}$$

$$\begin{aligned}
&= L_1(r) + \frac{1}{V} (2L_1(t)L_2(r) - VL_1(t) - 2L_1(r)L_2(r) + VL_1(r) + 2L_1(r)L_2(r) \\
&\quad - 2L_1(t)L_2(r)),
\end{aligned}$$

$$RTC(r) = 2L_1(r) - L_1(t).$$

Analogously,

$$\begin{aligned}
RAB(r) &= \int_0^r x dH_1(x) + \int_0^r x dH_2(x) \\
&= \int_0^t x dH_1(x) + rH_1(r) + \int_0^t x dH_2(x) + rH_1(r) \\
&= \frac{1}{V} \int_0^t x dL_2(x) + r(1 - H_1(t)) + \frac{1}{V} \int_0^t x dL_1(x) + r(1 - H_2(t)) \\
RAB(r) &= \frac{1}{V} \left( t(L_1(t) + L_2(t)) - \int_0^t (L_1(t) + L_2(t)) dx \right) + r(2 - H_1(t) - H_2(t)).
\end{aligned}$$

■